# Scalable many-light methods 

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## Instant radiosity

- Approximate indirect illumination by


## Virtual Point Lights (VPLs)

1. Generate VPLs
2. Render with VPLs


## Instant radiosity with glossy surfaces



- Large number of VPLs required
- True even for diffuse scenes
- Scalability issues


## Scalable many-light methods

1. Generate many, many VPLs
2. Use only the most relevant VPLs for rendering

- Choosing the right VPLs
- Per-pixel basis
- Lightcuts [Walter et al 05/06]
- Per-image basis
- Matrix Row Column Sampling [Hašan et al. 07]


## Scalability with many lights

## Approach \#1: Lightcuts \& Multi-dimensional lightcuts

Walter et al., SIGGRAPH 2005/06
Slides courtesy Bruce Walter:
http://www.graphics.cornell.edu/~bjw/papers.html

# Lightcuts: A Scalable Approach to Illumination 

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## Lightcuts

## - Efficient, accurate complex illumination



Environment map lighting \& indirect Time 111 s


Textured area lights \& indirect Time 98s
(640x480, Anti-aliased, Glossy materials)

## Scalable

- Scalable solution for many point lights
- Thousands to millions
- Sub-linear cost




## Complex Lighting

- Simulate complex illumination using point lights
- Area lights
- HDR environment má
- Sun \& sky light
- Indirect illumination
- Unifies illumination
- Enables tradeoffs between components

Area lights + Sun/sky + Indirect

## Lightcuts Problem






4
4

Visible<br>surface

$$
8
$$

## Lightcuts Problem



## Lightcuts Problem



## Key Concepts

- Light Cluster
- Approximate many lights by a single brighter light
(the representative light)



## Key Concepts

- Light Cluster
- Light Tree
- Binary tree of lights and clusters



## Key Concepts

- Light Cluster
- Light Tree
- A Cut
- A set of nodes that partitions the lights into clusters



## Simple Example



## Three Example Cuts



## Three Example Cuts



Good
Bad

## Three Example Cuts



## Three Example Cuts



## Algorithm Overview

- Pre-process
- Convert illumination to point lights
- Build light tree
- For each eye ray
- Choose a cut to approximate the illumination


## Convert Illumination

- HDR environment map
- Apply captured light to scene
- Convert to directional point lights using [Agarwal et al. 2003]
- Indirect Illumination
- Convert indirect to direct illumination using Instant Radiosity [Keller 97]
- Caveats: no caustics, clamping, etc.

- More lights = more indirect detail


## Algorithm Overview

- Pre-process
- Convert illumination to point lights
- Build light tree
- For each eye ray
- Choose a cut to approximate the local illumination
- Cost vs. accuracy
- Avoid visible transition artifacts


## Perceptual Metric

- Weber's Law
- Contrast visibility threshold is fixed percentage of signal
- Used 2\% in our results
- Ensure each cluster's error < visibility threshold
- Transitions will not be visible
- Used to select cut


## Illumination Equation

## result $=\sum_{\text {Ights }} M_{\mathrm{i}} G_{\mathrm{i}} V_{\mathrm{i}} I_{\mathrm{i}}$



Currently support diffuse, phong, and Ward

## Illumination Equation

## Illumination Equation



## Cluster Approximation

## result $\approx M_{\mathrm{j}} G_{\mathrm{j}} V_{\mathrm{j}} \sum_{\text {noghts }} I_{\mathrm{i}}$

$j$ is the representative light


## Cluster Error Bound

$$
\text { error } \leq M_{\mathrm{ub}} G_{\mathrm{ub}} V_{\mathrm{ub}} \sum I_{\mathrm{i}_{\mathrm{i} \text { ght }}}
$$

- Bound each term
- Visibility <= 1 (trivial)

- Intensity is known
- Bound material and geometric terms using cluster bounding volume


## Cut Selection Algorithm

- Start with coarse cut (eg, root node)



## Cut Selection Algorithm

- Select cluster with largest error bound



## Cut Selection Algorithm

- Refine if error bound > 2\% of total


Cut Selection Algorithm


Cut Selection Algorithm


Cut Selection Algorithm


## Cut Selection Algorithm

- Repeat until cut obeys 2\% threshold



Kitchen, 388K polygons, 4608 lights (72 area sources)


Error

Kitchen, 388K polygons, 4608 lights (72 area sources)

## Combined Illumination



Lightcuts 1285
4608 Lights
(Area lights only)


Lightcuts 2905
59672 Lights
(Area + Sun/sky + Indirect)

## Combined Illumination



Lightcuts 1285
4608 Lights
(Area lights only)
Avg. 259 shadow rays / pixel


Lightcuts 2905
59672 Lights
(Area + Sun/sky + Indirect)
Avg. 478 shadow rays / pixel (only 54 to area lights)


Error x 16


## Scalable

- Scalable solution for many point lights
- Thousands to millions
- Sub-linear cost




## Lightcuts



- Problem: Large cuts in dark areas


## Lightcuts Recap

- Unified illumination handling
- Scalable solution for many lights
- Locally adaptive representation (the cut)
- Analytic cluster error bounds
- Most important lights always sampled
- Perceptual visibility metric
- Problems
- Large cuts in dark regions
- Need tight upper bounds for BRDFs


# Multidimensional Lightcuts 

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## Problem

- Simulate complex, expensive phenomena
- Complex illumination
- Anti-aliasing
- Motion blur
- Participating media
- Depth of field


$$
\text { Pixel }=\iint_{\text {Time }} \int_{\substack{\text { Pixel Lights } \\ \text { Area }}} \int_{\mathbf{L},} \mathrm{L}(\mathbf{X}) \ldots
$$

## Problem

- Simulate complex, expensive phenomena
- Complex illumination
- Anti-aliasing
- Motion blur
- Participating media
- Depth of field


$$
\text { Pixel }=\iint_{\text {Volume Time }} \int_{\substack{\text { Pixel Lights } \\ \text { Area }}} \int(X, \omega) \ldots
$$

## Problem

- Simulate complex, expensive phenomena
- Complex illumination
- Anti-aliasing
- Motion blur
- Participating media
- Depth of field


$$
\text { Pixel }=\iint_{\text {Aperture }} \int_{\text {Volume }} \int_{\text {Time }} \int_{\text {Pixel Lights }} \int_{\text {Area }} \mathrm{L}(\mathbf{X}, \omega) \ldots
$$

## Problem

- Complex integrals over multiple dimensions
- Requires many samples



## Multidimensional Lightcuts

- Solves all integrals simultaneously
- Accurate
- Scalable





## Point Sets

- Discretize full integral into 2 point sets
- Light points (L)
- Gather points (G)



## Point Sets

- Discretize full integral into 2 point sets
- Light points (L)
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## Point Sets

- Discretize full integral into 2 point sets
- Light points (L)
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## Point Sets

- Discretize full integral into 2 point sets
- Light points (L)
- Gather points (G)



## Discrete Equation

- Sum over all pairs of gather and light points
- Can be billions of pairs per pixel

$$
\text { Pixel }=\sum_{(\mathrm{j}, \mathrm{i}) \in \mathbf{G \times L}} \mathrm{S}_{\mathrm{j}} M_{\mathrm{ji}} G_{\mathrm{ji}} V_{\mathrm{ji}} I_{\mathrm{i}}
$$

## Product Graph

- Explicit hierarchy would be too expensive
- Up to billions of pairs per pixel
- Use implicit hierarchy
- Cartesian product of two trees (gather \& light)


## Product Graph



## Product Graph



Gather tree

## Product Graph



Gather tree

## Product Graph

## Product Graph



## Product Graph

## Product Graph



## Product Graph



## Cluster Representatives



## Cluster Representatives



## Error Bounds

- Collapse cluster-cluster interactions to point-cluster
- Minkowski sums
- Reuse bounds from Lightcuts

- Compute maximum over multiple BRDFs
- Rasterize into cube-maps
- More details in the paper



## Algorithm Summary

- Once per image
- Create lights and light tree
- For each pixel
- Create gather points and gather tree for pixel
- Adaptively refine clusters in product graph until all cluster errors < perceptual metric


## Scalability

- Start with a coarse cut
- Eg, source node of product graph



## Scalability

- Choose node with largest error bound \& refine - In gather or light tree



## Scalability

- Choose node with largest error bound \& refine - In gather or light tree



## Scalability

- Repeat process



## Algorithm summary

- Until all clusters errors < perceptual metric
- 2\% of pixel value (Weber's law)



## Results

- Limitations
- Some types of paths not included
- Eg, caustics
- Prototype only supports diffuse, Phong, and Ward materials and isotropic media


## Roulette



7,047,430 Pairs per pixel Time 590 secs
Avg cut size 174 (0.002\%)

## Scalability

## Image time vs. Gather points



## Metropolis Comparison



## Kitchen



5,518,900 Pairs per pixel Time 705 secs
Avg cut size 936 (0.017\%)


180 Gather points X 13,000 Lights = 234,000 Pairs per pixel Avg cut size 447 (0.19\%)

114,149,280 Pairs per pixel Avg cut size 821 Time 1740 secs

## Scalability with many lights

# Approach \#2: Matrix Row-Column sampling 

Hašan et al., SIGGRAPH 2007

Slides courtesy Miloš Hašan:
http://www.cs.cornell.edu/~mhasan/

## Improving Scalability and Performance

 Bruteforce:


10 min
$\downarrow$

3.8 sec


13 min
$\downarrow$

13.5 sec


20 min
$\downarrow$

16.9 sec

## A Matrix Interpretation



## Problem Statement

- Compute sum of columns

Lights


## Low-Rank Assumption

- Column space is (close to) low-dimensional



## Ray-tracing vs Shadow Mapping




## Computing Column Visibility

- Regular Shadow Mapping



## Row-Column Duality

- Rows: Also Shadow Mapping!



## Image as a Weighted Column Sum

- The following is possible:


Compute small subset of columns

compute weighted sum

- Use rows to choose a good set of columns!


## The Row-Column Sampling Idea


compute rows $\begin{gathered}\text { chowstocchonsse } \\ \text { Galuwenghtr } \\ \text { weights? }\end{gathered} \quad \begin{gathered}\text { compute columns }\end{gathered} \begin{gathered}\text { weighted } \\ \text { sum }\end{gathered}$

## Clustering Approach



## Reduced Matrix



## Weights and Information Vectors

- Weights $\mathrm{w}_{\mathrm{i}}$
- Norms of reduced columns
- Represent the "energy" of the light
- Information vectors $\mathrm{X}_{\mathrm{i}}$
- Normalized reduced columns
- Represent the "kind" of light's contribution


## Visualizing the Reduced Columns

Reduced columns:
vectors in highdimensional space

## Monte Carlo Estimator

- Algorithm:

1. Cluster reduced columns
2. Choose a representative in each cluster, with probability proportional to weight
3. Approximate other columns in cluster by (scaled) representative

- This is a Monte Carlo estimator
- Which clustering minimizes its variance?


## The Clustering Objective

- Minimize:

- where: $\operatorname{cost}(C)=\sum w_{i} w_{j}\left\|\mathbf{x}_{i}-\mathbf{x}_{j}\right\|^{2}$

cost of a cluster

$$
i, j \in C
$$

sum over all pairs in it

weights between information vectors

## Clustering Illustration



## How to minimize?

- Problem is NP-hard
- Not much previous research
- Should handle large input:
- 100,000 points
- 1000 clusters
- We introduce 2 heuristics:
- Random sampling
- Divide \& conquer


## Clustering by Random Sampling


$\longleftarrow$ Very fast (use optimized BLAS)

- Some clusters might be too small / large


## Clustering by Divide \& Conquer



Splitting small clusters is fast

- Splitting large clusters is slow


## Combined Clustering Algorithm



## Combined Clustering Algorithm



## Full Algorithm



Assemble rows into reduced matrix


Cluster reduced columns

Compute rows (GPU)


Choose representatives


Compute columns (GPU)


Weighted sum

## Example: Temple

- 2.1m polygons
- Mostly indirect \& sky illumination
- Indirect shadows


Our result: $16.9 \mathrm{sec}(300$


Reference: 20 min (using all 100k lights)

## Example: Kitchen

- 388k polygons
- Mostly indirect illumination
- Glossy surfaces
- Indirect shadows


Our result: 13.5 sec + 864 columns)
(432 rows


Reference: $13 \mathrm{~min} \quad$ (using all 100k lights)

## Example: Bunny

- 86gk polygons
- Incoherent geometry
- High-frequency lighting
- Kajiya-Kay hair shader


Our result: 3.8 sec
(100 rows
Reference: 10 min
(using all

+ 200 columns) 100k lights)

